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TWO PHASE FLOW THROUGH VERTICAL CAPILLARIES—EXISTENCE OF A STRATIFIED FLOW PATTERN

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Abstract—Two phase downflows were investigated in vertical capillaries with internal diameters in the range 0.5–7.1 mm. Flow pattern regime maps were developed for water-air, distillate-air and water-distillate systems. Over the range of flowrates studied, stratified flow was found to occur in the smaller diameter (<3 mm) capillaries where interfacial tension forces dominate. A modified Hagen-Poiseuille model, which incorporated surface tension effects and which employed the concept of an effective hydraulic diameter, was developed for each of the phases. These equations were used to predict pressure drop and holdup for laminar flow of the two phases in the capillaries. Comparison of pressure drop predictions and measurements gave an RMS error of 14% for the water-distillate system and 8% for the water-air system.

INTRODUCTION

Two phase downward flows of gas-liquid and liquid-liquid mixtures in narrow tubes introduce two complications to traditional multiphase flow processes. In comparison to work carried out on horizontal or upward tube flows, two phase downflow is a relatively understudied area. Additionally, the role of interfacial tension becomes more significant as tube diameter decreases. The latter effect is well recognised in two phase flow through small diameter particles, where previous work has shown that the two phases can be considered to flow in a bundle of tortuous capillaries the extent and orientation of which are determined by liquid bridges between the particles. In turn, these liquid bridges are stabilized by surface tension forces which predominate in beds of small size particles, e.g. <2 mm (Vuckovic 1976, Kan & Greenfield 1978, Bemer & Zuiderweg 1978). Two phase downflow in vertical capillaries represents an idealisation of this situation.

Table 1 provides a summary of previous work on two-phase vertical downward flows. Reported flow regimes in such systems are depicted in figure 1 and summarized below.

(a) Dispersed or Bubble Flow: The nonwetting phase (gas) is dispersed in a continuum of the wetting phase (liquid).
(b) Slug or Plug Flow: Bubble coalescence occurs and bubble diameter progressively approaches that of the tube.
(c) Annular Flow: Liquid or wetting phase flows down the wall of the tube as a film while the gas flows as a cone in the centre of the tube.
(d) Droplet Flow: Liquid or wetting phase is discontinuous with the gas or nonwetting phase continuous

It is noteworthy that there have been no reports of stratified flow structures in two phase vertical flows. Barnea *et al.* (1980, 1982a) and Spedding & Nguyen (1980) have studied flow regimes in inclined tubes (ca. 2.5 cm diameter). As the angle of downward inclination increased from 0° (i.e. horizontal) to 80°, they found the flow regimes to be identical to those for horizontal flow which include smooth stratified and wavy stratified flow. For 90° inclination (vertical), however, only slug, annular and dispersed regimes were observed. Wallis (1969) argues that stratified flow will not occur in vertical tubes because of complications caused by the action of gravity. His argument, however, does not consider the effect of forces due to interfacial tension because it was aimed at flows in large diameter

Table 1. Previous studies on two-phase vertical downward flows

Author	System	Pipe diameter	Regimes
Golan & Stenning (1969)	Air-water	31.8 mm	Slug, bubble, annular
Martin (1973)	Air-water	140 mm	Bubble, slug
Martin (1976)	Air-water	—	Slug
Kulov et al. (1979)	Air-water	25 mm	Unclear
Spedding & Nguyen (1980)	Air-water	45.5 mm	Slug, annular, bubble droplet
Barnea et al. (1982b)	Air-water	2-5, 51 mm	Slug, annular, dispersed

tubes. The effect of surface tension was shown by Benjamin (1957) who argued from wave theory that a falling film of a wetting phase will be unstable in a two-phase flow situation unless surface tension forces predominate.

This neglect of the role of interfacial tension simply reflects the relatively small amount of work carried out on two-phase flow in narrow tubes. Marschessault & Mason (1962) studied the flow of air bubbles through a capillary tube; Sue & Griffith (1964) studied gas-liquid horizontal flow in 1 mm diameter tubes, operating in the slug flow regime; Oya (1971 a, b) investigated flow regimes in tubes of 2, 3 and 6 mm diameter, at the same time developing semi-empirical equations for pressure drop. Recently, Barnea *et al.* (1983) reported on two-phase horizontal and upward flows in tubes of 4.0, 6.0, 8.15, 9.85 and 12.3 mm diameter. They found that the models of Taitel & Dukler (1976) and Taitel *et al.* (1980) for predicting the flow transition boundaries were inaccurate when applied to small diameter tubes because of the neglect of surface tension forces.

EXPERIMENTAL EQUIPMENT AND PROCEDURE

The equipment (figure 2) was designed to allow the controlled entry of two fluid phases into a capillary. Glass capillaries of sizes 0.5, 1.0, 1.3, 2.2, 3.1, 4.2, 5.0 and 7.1 ($\pm$ 0.1) mm internal diameter were used in the experiments. Each capillary consisted of a 60 cm tube, and a pressure tapping with a forked endpiece to allow entry of fluids. Pressure drops in the range 0–40 cm Hg were measured using a manometer while a Bourdon tube gauge was used for higher pressures. Saturated air was supplied through a sparger. Liquids were supplied from pressure vessels pressurized using line air pressure at 210 kPa. Three systems were studied:

- water-air
- distillate-air
- water-distillate

At 25°C, the distillate had a density of 720 kg/m^3, a viscosity of 0.82×10^{-3} Pa.s and a surface tension of 33×10^{-3} newtons/m. Initial experiments were aimed at the identification of flow regimes. A Tessovar camera (X20) was used to photograph the different flow regimes

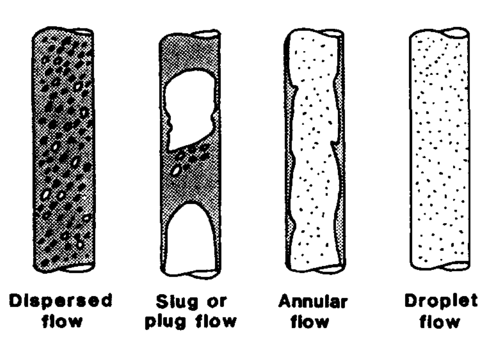

Figure 1. Flow regimes found in previous work on two-phase vertical downward flows.

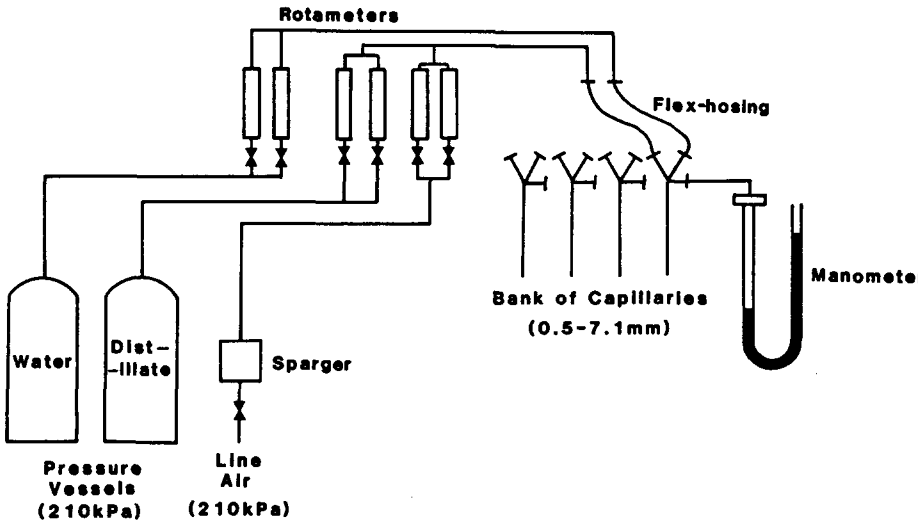

Figure 2. Schematic of experimental equipment for two-phase capillary flow.

which developed as the flowrates of the wetting and non-wetting phases were altered. Regime transitions were noted visually by setting the flowrate of one phase and increasing or decreasing that of the second phase. Additional experiments were undertaken to measure the holdup and pressure drop across a fixed capillary length for different flow rates of both wetting and non-wetting phases. All measurements were made at a constant temperature of 25°C. Holdup was measured by simultaneously blocking the ends of the capillary under steady flow conditions, after which the masses of each phase in the capillary could be determined.

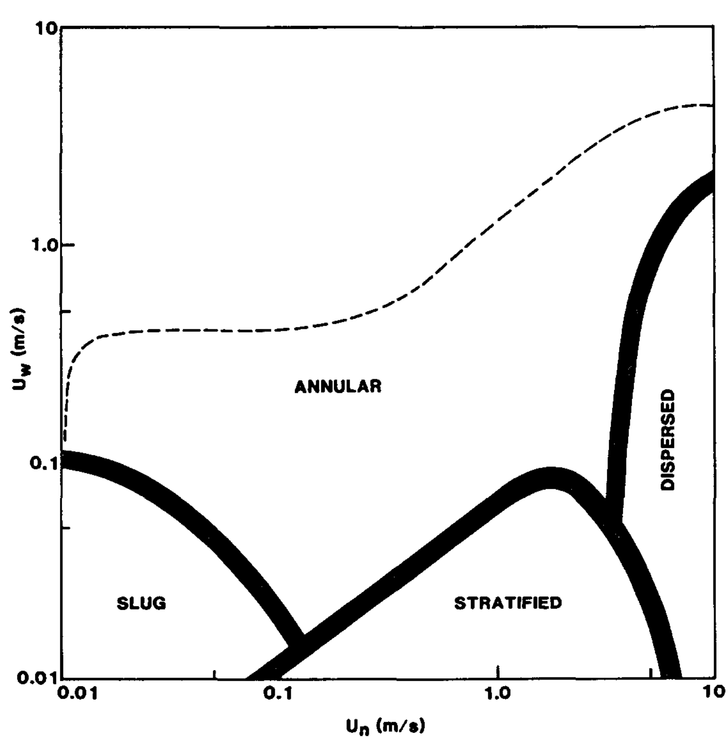

Figure 3. Flow regime diagram for cocurrent downflow of water-air and distillate-air in vertical capillaries.

DESCRIPTION OF FLOW REGIMES

The flow pattern map of Mandhane *et al.* (1974) was used to depict the data because of its widespread acceptance in flow regimes which occur for two-phase flow in larger diameter tubes. The flow regime maps are shown in figures 3 and 4. In these, the boundary regions between different flow regimes are shaded; they were determined by visual observation. The water–air and distillate–air data of figure 3 were determined from capillaries of 1.3 mm, 2.2 mm and 3.1 mm diameter. While stratified flow could be seen at the smaller diameters, the magnification system was not sufficiently powerful to allow the results to be plotted reliably. The water-distillate data of figure 4 were determined from capillaries of 1.0 mm, 1.3 mm and 2.2 mm diameter.

The following stable regimes were observed:

Water-air, distillate-air: stratified
dispersed
slug
annular

Water-distillate: stratified
dispersed
slug

Photographs of the stratified flow regime are shown in figure 5 for the oil-water system. Figure 5b is a side-on shot of the flow shown in figure 5a. Two distinct phases can be seen meandering down the capillary wall. Figure 5c shows a magnified view of the curved fluid-fluid interface while figure 5d depicts the transition from stratified to dispersed flow. The dark vertical boundaries in each of the photographs represent the outside walls of the capillary. It is believed that this is the first report of a stratified flow regime in vertical two phase cocurrent flow.

The annular flow regime was found to be unstable for the water-distillate system. It occurred only as a transient state between the stratified and dispersed flow regimes. At high flow rates, the equipment was limited by the allowable pressure. Hysteresis occurred at the limits of stability, i.e. if the non-wetting phase flowrate was increased to de-stabilize a stable stratified flow structure, the point of breakdown was different from that found when the nonwetting phase flowrate was decreased to stabilize a stratified structure from a dispersed type flow, the difference in flowrates being up to 70%. Clearly, a stable stratified flow

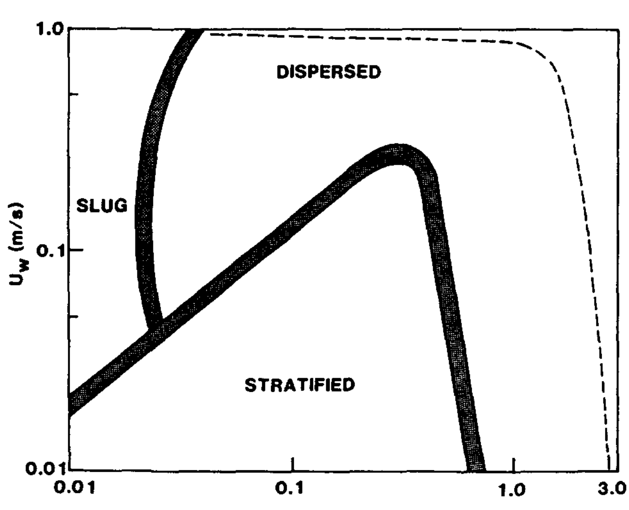

Figure 4. Flow regime diagram for cocurrent water-distillate downflow in vertical capillaries.

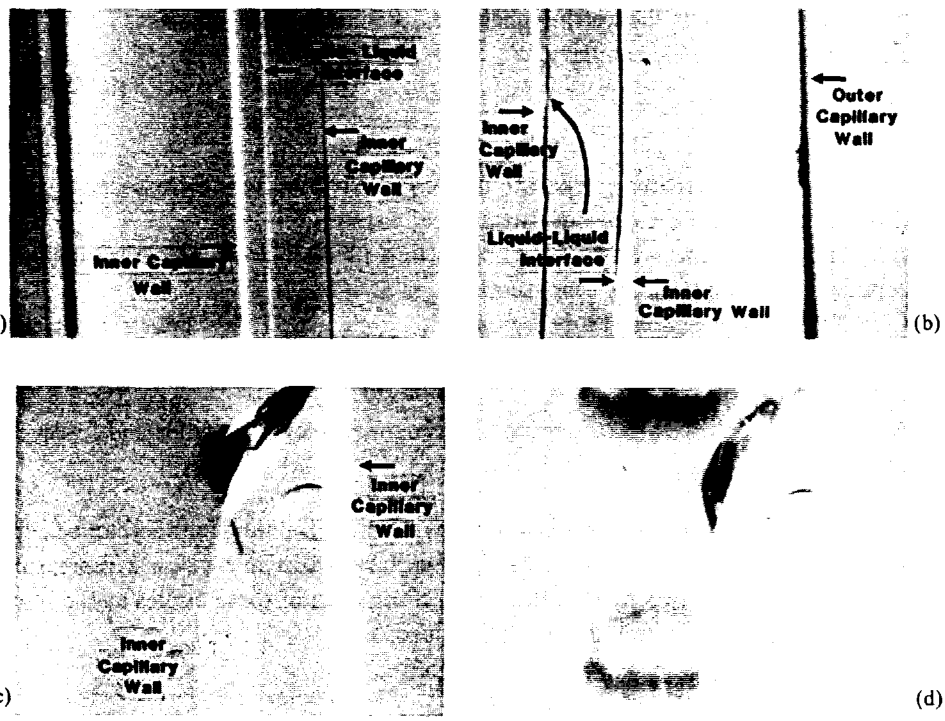

Figure 5. Photographs of stratified flow for cocurrent downward flow in capillaries. (a) Stratified flow, diesel-water system; (b) Stratified flow, side-on shot of figure 5a; (c) Stratified flow, curved liquid-liquid interface; (d) Transition from stratified flow to dispersed flow.

structure is difficult to breakdown. The breakdown points of the stratified flow structure were not reproducible when this regime was broken down by increasing the nonwetting phase flowrate. Hence, the flow regime diagrams were generated by starting with a dispersed type of flow and decreasing the nonwetting phase flowrate for a fixed wetting phase flowrate. This gave much better reproducibility, with estimated errors being of the order of 10–15%; the points of transition were observed visually. While stratified flow was observed only with capillaries of diameter <3.1 mm, these errors and the limitations of the equipment prevented the specific dependence of stratified flow on capillary diameter from being determined.

A STRATIFIED FLOW MODEL FOR TWO-PHASE CO-CURRENT FLOW IN VERTICAL CAPILLARIES

The proposed model is based on an extension of those of Martinelli *et al.* (1946) and of Vuckovic (1976) to account for surface tension effects. For stratified flow to occur in vertical capillaries, surface tension forces must be dominant, hence, such effects should be included in any analysis.

In developing a model for vertical stratified flow, the following assumptions were made:

(i) The wetting and nonwetting phases are divided by a surface which is mobile and which moves in a direction perpendicular to the flow direction. The shape of the interface depends on the wettability characteristics of the system and on the size of the flow channel.

(ii) The analysis applies to any pair of wetting and nonwetting phases, although a liquid and gas will be used in the derivation.

(iii) Flow is in the laminar regime for each phase.

(iv) Phases are immiscible with the holdup or saturation of a phase being defined by:

$$S_i = \frac{\text{volume of fluid phase } i \text{ in length } L \text{ of capillary}}{\text{total volume in length } L \text{ of capillary}}, \qquad [1]$$

where i is w or n, w is the wetting phase, and n is non-wetting phase.

For the case of a single capillary,

$$S_w + S_n = 1. \qquad [2]$$

(v) Since each phase creates a separate flow channel of uniform cross-section, entrance flowrate must equal the exit flowrate at steady state.

(vi) At steady state, the wetting and nonwetting phase holdup or saturation in each capillary is the same at any cross-section.

(vii) Liquid and gas pressures at any cross-section are equal. This is not strictly true since the gas pressure will be slightly higher due to the equilibrium capillary pressure (Vuckovic 1976).

(viii) As discussed earlier, surface tension forces dominate over gravitational forces in fine capillaries.

The fluid distribution at any cross-section for any ratio of fluid flowrates can be depicted as shown in figure 6. Circle 1 represents the capillary wall while circle 2 represents the fluid-fluid interface. Tangents may be drawn touching each of these circles at the interface of the two fluids. From the geometrical relationships:

$$S_w = \left(1 - \frac{a}{\pi}\right) - \frac{x^2}{y^2}\left(1 - \frac{b}{\pi}\right), \qquad [3]$$

where S_w is holdup of wetting phase, a is $\frac{1}{2}(\pi/180\,\alpha - \sin\alpha)$, b is $\frac{1}{2}(\pi/180\,\beta - \sin\beta)$, x is $2\sin\alpha/2$, y is $2\sin\beta/2$, α is $\beta - 2\theta$, θ is apparent wettability angle.

A simplified approach to predicting stratified flow pressure drop is developed in this section. The approach assumes that the dominant effect of surface tension forces is to alter the geometry of the flow channels, thereby affecting the momentum balance for each phase. Each flow channel is considered as a separate, noncircular capillary inside a circular

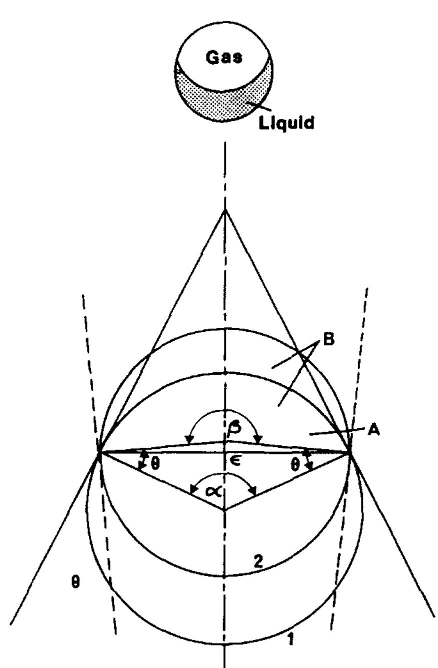

Figure 6. Geometrical relationships for wetting fluid and nonwetting fluid flow channels in a capillary.

capillary of diameter d'. The flow channels are reduced to equivalent circular pipes with the hydraulic diameter of each being given by four times the actual flow area divided by the perimeter of the flow channel. In this way, the Hagen-Poiseuille equation is applied to each non-circular capillary.

A more rigorous approach would follow the analysis of Taitel & Dukler (1976) and Chen & Spedding (1983) with appropriate modifications to account for the effects of interfacial tension. Such a procedure would lead to a complex but more rigorous solution than what is proposed, but in the absence of information on the hydrodynamics at the phase interface, has not been attempted.

It is recognised that the proposed approach effectively lumps momentum transfer at the capillary wall and that at the wetting-nonwetting fluid interface. For a gas phase, this is not particularly limiting since the assumption is often made, in models of two phase smooth stratified flow, that the shear stress at the gas-liquid interface is equivalent to that at the pipe wall (Taitel & Dukler 1976; Chen & Spedding 1983). Wave action at the interface modifies this assumption and a dependence on wetting phase holdup is found (Wallis 1970). The simplified approach is less reasonable when applied to the liquid phase; it is, however, necessary to allow for the effective lumping of the boundary momentum transfer terms. It is additional information on the state of the interface which will allow a more precise model to be written (van Swaaij 1983).

Using the above approach, the following relationships are developed:

$$D_{hw} = \frac{d'S_w}{1 - M} = \frac{4 \times \text{flow area of wetting phase}}{\text{wetting phase channel perimeter}}, \qquad [4]$$

$$D_{hn} = \frac{d'S_n}{N} = \frac{4 \times \text{flow area of nonwetting phase}}{\text{nonwetting phase channel perimeter}}, \qquad [5]$$

$$q_w = \frac{d'^4 S_w^3 \pi (P_1 - P_2)}{128(1 - M)^2 \mu_w L}, \qquad [6]$$

$$q_n = \frac{d'^4 S_n^3 \pi (P_1 - P_2)}{128 N^2 \mu_n L}, \qquad [7]$$

where d' is capillary diameter, D_{hw}, D_{hn} is hydraulic diameter of wetting and nonwetting channels respectively, L is capillary length.

$$M = \frac{\alpha}{360°}\left(1 + \frac{x}{y}\right) - \frac{x}{y}\left(1 - \frac{2\theta}{360}\right), \qquad [8]$$

$$N = \frac{\alpha}{360}\left(1 - \frac{x}{y}\right) + \frac{x}{y}\left(1 - \frac{2\theta}{360}\right), \qquad [9]$$

where P_1, P_2 is pressure at beginning and end of capillary of length L, q_i is volumetric flowrate of fluid phase i through capillary; μ_i is viscosity of fluid phase i.

In [6] and [7], the gravitational driving forces have been neglected. The actual driving force should be $[(P_1 - P_2)/L + S_w \rho_w g + S_n \rho_n g]$. The gravitational terms have been omitted to simplify the subsequent development. Their addition, however, is straightforward. For 1 mm diameter capillaries, errors of up to 10% will occur if no correction is made for gravity, with this decreasing for lower saturation values in gas-liquid systems and for finer capillaries.

The above model allows calculation of the system pressure drop and holdup of each of the flowing phases if the apparent wettability angle is known. The apparent wettability angle is a function of the types of fluids flowing, capillary material (i.e. glass, metal), and the pressure and temperature of the system. Although it must be determined experimentally, its value will be approximately constant over a wide range of operating conditions for a given capillary material (e.g. glass) and for a given set of fluids; hence, its value can be determined independently and then utilised to predict pressure drop and holdup.

To estimate θ, the definition of relative permeability is introduced:

$$K_{rw} = K_w/K, \qquad [10]$$

$$K_{rn} = K_n/K \qquad [11]$$

where K_{rw}, K_{rn} are the relative permeabilities of the wetting and nonwetting phases, respectively; K_w, K_n are the effective permeabilities (i.e. based on total flow area) of the wetting and non-wetting phases, respectively, under two phase flow conditions, i.e.

$$K_w = \frac{q_w \mu_w}{A} \frac{L}{(P_1 - P_2)}, \qquad [12]$$

$$K_n = \frac{q_n n}{A} \cdot \frac{L}{(P_1 - P_2)}, \qquad [13]$$

where A is capillary cross-section area; and K is the absolute permeability of the capillary to a single-phase as defined by Darcy's Law (Dullien 1979).

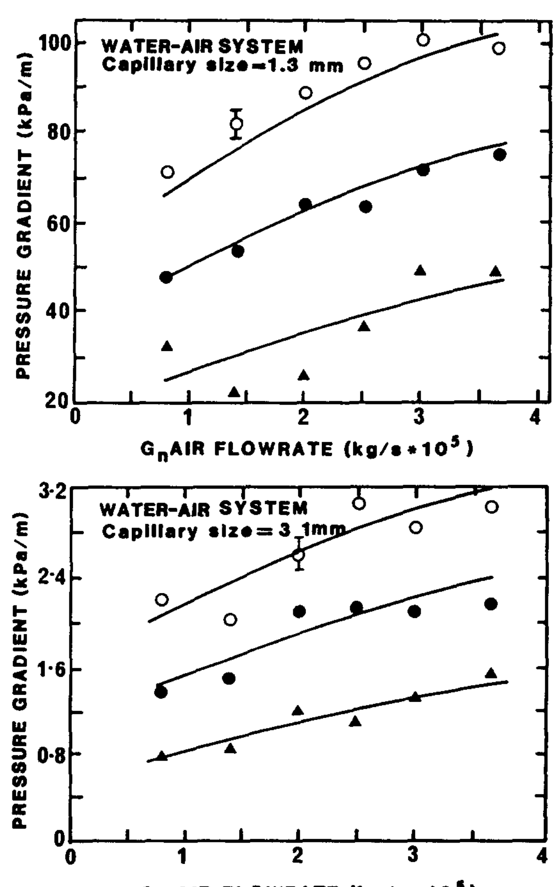

Figure 7. Prediction of capillary pressure drop for cocurrent water-air downflow—stratified flow regime. (▲ . . . G_w = 5.4×10^{-4} kg/s; ● . . . G_w = 1.3×10^{-3} kg/s; ○ . . . G_w = 2.1×10^{-3} kg/s; solid line represents model predictions).

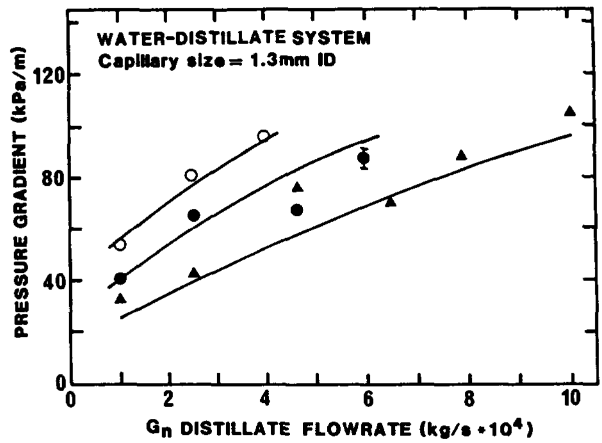

Figure 8. Prediction of capillary pressure drop for cocurrent water-distillate downflow—stratified flow regime. (▲ ... $G_w = 3.33 \times 10^{-4}$ kg/s; ● ... $G_w = 7.33 \times 10^{-4}$ kg/s; O ... $G_w = 11.25 \times 10^{-4}$ kg/s; solid line represents model predictions).

For single phase flow in a capillary, application of the Poiseuille equation leads to:

$$K = \frac{d'^2}{32}. \quad [14]$$

From [6], [7], [10–14] it can be shown that:

$$K_{rw} = \frac{S_w^3}{(1 - M)^2}, \quad [15]$$

$$K_{rn} = \frac{S_n^3}{N^2}. \quad [16]$$

Measurement of pressure drop and holdup for a particular set of flowrates leads via equations [10], [12], [14] and [15] to an expression for M in terms of measured variables. The holdup measurement provides a value of S_w. Simultaneous solution of the non-linear equations [3] and [8] leads to values of the angle α and the wettability angle, θ.

With the wettability angle for a pair of fluids known, the model equations can be solved to predict pressure drops and holdups for stratified two phase downward flow in capillaries.

From equations [6] and [7],

$$\frac{q_w}{q_n} = \frac{S_w^3 N^2 \mu_n}{S_n^3 (1 - M]^2 \mu_w}, \quad [17]$$

where $S_w + S_n = 1$ for single capillaries.

The simultaneous solution of [3] and [17] leads to values of the wetting phase holdup S_w and angle α. In turn, the pressure gradient can be predicted from [11], [13] and [16].

The values of apparent wettability angles were found experimentally using the method outlined above, to be as follows: water-air = 40 ± 5 degrees; water-distillate = 30 ± 15 degrees. The large uncertainty in the water-distillate system estimate was caused by the need to separate, completely, relatively small volumes of liquids to determine the holdup of each phase. A sensitivity analysis of the model equations shows that the effect of changes in the wettability angle, θ, is moderate, with a 50% variation in θ causing at most a 13% change in pressure drop.

Results from using the model to predict pressure drop for stratified flow in vertical capillaries are shown in figure 7 for the water-air system and figure 8 for the water-distillate system. The error bars on each diagram indicate the maximum standard deviation from multiple readings. Average deviations from experimental points for the water-air and water-distillate systems were found to be 8 and 14% respectively. The model shows better agreement with the data than that of Martinelli *et al.* (1946) for horizontal stratified flow which has an uncertainty of ±50%. We suggest that the better ability of model to predict pressure drop is due to the incorporation of surface tension effects in the model. The analysis is being extended to predict the pressure drop in two phase flow through fine porous media.

CONCLUSIONS

(1) Stratified flow was found to occur in capillaries with internal diameter <3 mm; it is believed this is the first such report of a stratified flow regime in two phase concurrent downflow.

(2) A model was developed to predict pressure drop and holdup (analogous to saturation) for stratified flow in vertical capillaries of small diameter, where surface tension forces are thought to dominate gravitational forces.

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